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Technical Memorandum

NECESSARY AND SUFFICIENT OBSERVABILITY CONDITIONS FOR BEARINGS-ONLY TARGET MOTION ANALYSIS

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ABSTRACT

The observability requirements for bearings-only target motion analysis (TMA) are rigorously established by solving a third-order nonlinear differential equation. Closed-form expressions are developed and subsequently used to specify necessary and sufficient conditions on own ship motion that ensure a unique tracking solution. It is shown that for certain types of maneuvers the estimation process remains unobservable, even when the associated bearing rate is non-zero. Such maneuvers are frequently overlooked in heuristic discussions of TMA observability, which may account for some common misconceptions regarding the characteristics of acceptable own ship motion.

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INTRODUCTION

Passive bearings-only tracking techniques are utilized in a variety of theoretical and practical applications.[1-5] In the ocean environment, two-dimensional target motion analysis (TMA) is perhaps most familiar.[4-7] Here, a moving observer (own ship) monitors sonar bearings from an acoustic source (target) travelling with constant velocity, and subsequently processes these measurements to obtain estimates of source position and velocity. The geometric configuration is depicted in figure 1, where both own ship and target are presumed to lie in the same horizontal (x-y) plane.

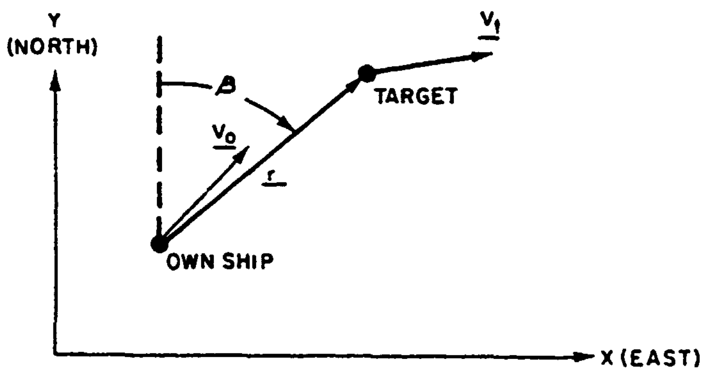

Figure 1

Whenever bearing data are extracted from a <u>single</u> sensor, the aforementioned estimation problem will remain unobservable prior to an own ship maneuver. Indeed, unique tracking solutions cannot be obtained for unaccelerated motion.[7] It is this prerequisite maneuver which distinguishes bearings-only TMA from more conventional localization techniques (e.g., classical triangulation ranging, etc.) and introduces added complexity to the problem.

Unfortunately, the subject of TMA observability has not received adequate attention in the literature. Solution uniqueness requirements have evolved mainly from heuristic and/or geometric arguments, which often lack mathematical rigor. As a result, certain misconceptions prevail regarding the characteristics of acceptable own ship motion; e.g., the assumption that any maneuver will suffice provided the associated bearing rate is non-zero. In this paper we rigorously establish necessary and sufficient conditions for TMA observability. The system is recast in equivalent linear form which allows direct application of a simplified observability test. Subsequent analysis yields a third-order nonlinear differential equation embodying the pertinent constraints on own ship motion. By solving this equation it is shown that certain types of maneuvers are unacceptable, despite the presence of non-zero bearing rates.

PROBLEM FORMULATION AND ANALYSIS

The equations of motion for a constant velocity target, as depicted in figure 1, may be expressed in the form

$$\dot{\underline{r}}(t) = \underline{v}(t) \tag{1a}$$

$$\dot{\underline{v}}(t) = -\underline{a}_o(t) \tag{1b}$$

where

$$\underline{v}(t) = \underline{v}_t(t)-\underline{v}_o(t). \tag{2}$$

Here, $\underline{r} = [r_x,r_y]'$ and $\underline{v} = [v_x,v_y]'$ represent the relative range and velocity, while $\underline{a}_o = [a_{ox},a_{oy}]'$ describes own ship acceleration. The remaining vectors $\underline{v}_o = [v_{ox},v_{oy}]'$ and $\underline{v}_t = [v_{tx},v_{ty}]'$ denote own ship and target velocity, respectively; however, these will not be used explicitly in the ensuing analysis. Finally, we note that equation set (1) can be integrated to yield the familiar expressions:[6]

$$\underline{r}(t) = \underline{r}(t_o)+(t-t_o)\underline{v}(t_o) - \int_{t_o}^{t} (t-\tau)\underline{a}_o(\tau)d\tau \tag{3a}$$

$$\underline{v}(t) = \underline{v}(t_o) - \int_{t_o}^{t} \underline{a}_o(\tau)d\tau \tag{3b}$$

where t_o denotes some arbitrary fixed initial time.

Since we are primarily concerned with the theoretical aspects of observability, only noise-free data measurements will be considered. For bearings-only TMA, these measurements consist of line-of-sight angles (see figure 1) which satisfy the nonlinear relation

$$\beta(t) = \tan^{-1}[r_x(t)/r_y(t)]. \tag{4}$$

Ordinarily, the presence of measurement nonlinearities would dictate an analysis utilizing techniques such as the "ratio test"[8]

or the "strongly positive semi-definite condition."[9] However, application of these relatively complicated procedures may be avoided here by rewriting (4) in equivalent linear form; i.e.,

$$M(t)\underline{x} = y(t) \tag{5}$$

where

$$\underline{x} = [r_x(t_o), r_y(t_o), v_x(t_o), v_y(t_o)]' \tag{6}$$

$$M(t) = [\cos\beta(t), -\sin\beta(t), (t-t_o)\cos\beta(t), -(t-t_o)\sin\beta(t)] \tag{7}$$

and

$$y(t) = \int_{t_o}^{t} (t-\tau)[a_{ox}(\tau)\cos\beta(t) - a_{oy}(\tau)\sin\beta(t)]d\tau . \tag{8}$$

Observe that $M(t)$ and $y(t)$ depend only upon $\beta(t)$ and $\underline{a}_o(t)$ which are known functions of time. The TMA solution parameters (unknown initial states) have all been incorporated into a constant vector $\underline{x}$. Consequently, (5) may be viewed as the time-varying measurement equation for a static linear system. It is well-known[10-12] that systems of the aforementioned type are completely observable if and only if the Grammian matrix

$$D(t) \triangleq \int_{t_o}^{t} M'(\tau)M(\tau)d\tau \tag{9}$$

is positive definite for some $t>t_o$.

Although (9) may be utilized to specify own ship motion requirements, the attendant computations are particularly cumbersome. A simpler approach[12] involves repeated differentiation of (5) to obtain a consistent set of linear equations for the unknown vector $\underline{x}$. Specifically,

$$A(t)\underline{x} = \underline{y}(t) \tag{10}$$

where A(t) is a (4x4) partitioned matrix

$$A(t) = \begin{bmatrix} M(t) \\ \hline \dot{M}(t) \\ \hline \ddot{M}(t) \\ \hline \dddot{M}(t) \end{bmatrix} \tag{11}$$

and

$$\underline{y}(t) = [y(t), \dot{y}(t), \ddot{y}(t), \dddot{y}(t)]' . \tag{12}$$

The general solution to (10) may be written in the form[13]

$$\underline{x} = A^{\#}(t)\underline{y}(t) + [I - A^{\#}(t)A(t)]\underline{z}(t) \tag{13}$$

where $A^{\#}(t)$ denotes the generalized inverse of A(t) and $\underline{z}(t)$ represents any arbitrary vector of appropriate dimension. From this result it is evident that $\underline{x}$ can be uniquely determined if and only if Rank[A(t)] = 4 for some $t > t_o$. Indeed, when A(t) attains full rank, the relation $A^{\#}(t) = A^{-1}(t)$ is automatically satisfied[13] so that

$$\underline{x} = A^{-1}(t)\underline{y}(t). \tag{14}$$

However, if A(t) remains rank deficient for all t, the second term in (13) will never vanish; consequently, $\underline{x}$ always contains an arbitrary component which renders the system unobservable.

The preceding discussion reveals that all solution uniqueness requirements are implicitly embodied in the scalar constraint relation $\det[A(t)] \not\equiv 0$ which ensures that A(t) will attain full rank for some value of t. Utilizing (7) and (11), this constraint

relation may be written as:*

$$\det[A(t)] = 2\dot{\beta}(t)\dddot{\beta}(t) - 3\ddot{\beta}^2(t) + 4\dot{\beta}^4(t) \not\equiv 0 \tag{15}$$

or, equivalently,

$$\beta(t) \not\equiv \tan^{-1}\left[\frac{r_x(t_o) + (t-t_o)v_x(t_o)}{r_y(t_o) + (t-t_o)v_y(t_o)}\right]. \tag{16}$$

Recall, however, that $\beta(t)$ must satisfy the relation

$$\beta(t) = \tan^{-1}\left[\frac{r_x(t_o) + (t-t_o)V_x(t_o) - \int_{t_o}^{t}(t-\tau)a_{ox}(\tau)d\tau}{r_y(t_o) + (t-t_o)V_y(t_o) - \int_{t_o}^{t}(t-\tau)a_{oy}(\tau)d\tau}\right] \tag{17}$$

which follows from (3a) and (4). Consequently, a unique tracking solution can be extracted from bearings-only data if and only if the right hand sides of (16) and (17) are functionally dissimilar; i.e.,

$$\left[\frac{r_x(t_o) + (t-t_o)V_x(t_o) - \int_{t_o}^{t}(t-\tau)a_{ox}(\tau)d\tau}{r_y(t_o) + (t-t_o)V_y(t_o) - \int_{t_o}^{t}(t-\tau)a_{oy}(\tau)d\tau}\right] \not\equiv \left[\frac{r_x(t_o) + (t-t_o)V_x(t_o)}{r_y(t_o) + (t-t_o)V_y(t_o)}\right]. \tag{18}$$

Examination of (18) readily reveals the (prerequisite) need for an own ship maneuver to render the system observable. Indeed, if no maneuvers occur, the constraint relation will be violated since both sides of (18) become identically equal. Although this result is well-known,[4-7] it does not constitute a complete statement of TMA observability. The reason is that (18) cannot be satisfied simply by

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* These results are derived by explicitly evaluating det[A(t)] and subsequently solving the homogeneous differential equation corresponding to det[A(t)] = 0 with appropriate initial conditions.

virtue of an arbitrary own-ship maneuver. The condition $\underline{a}_o(t) \not\equiv \underline{0}$ is necessary, but not sufficient; i.e., certain types of maneuvers are unacceptable. To rigorously fulfill all requirements embodied in (18), it is both necessary and sufficient that

$$\int_{t_o}^{t} (t-\tau)\underline{a}_o(\tau)d\tau \not\equiv \alpha(t)[\underline{r}(t_o) + (t-t_o)\underline{v}(t_o)] \qquad (19)$$

for any arbitrary scalar function $\alpha(t)$.

Equation (19) provides a comprehensive mathematical representation of the TMA observability requirements. Moreover, this expression is amenable to simple physical interpretation. For example, setting $\alpha(t) \equiv 0$ readily demonstrates how system observability is contingent upon an own ship maneuver as previously discussed. Similarly, if $\alpha(t) \not\equiv 0$, it becomes evident that the motion constraints imposed by (19) can be violated even though $\underline{a}_o(t) \not\equiv \underline{0}$; thus, certain types of maneuvers are precluded. These particular maneuvers always result in own ship positional changes which lie along the instantaneous bearing line (see figure 2). As such, the time characteristics of $\beta(t)$ will become indistinguishable from those produced by unaccelerated motion.

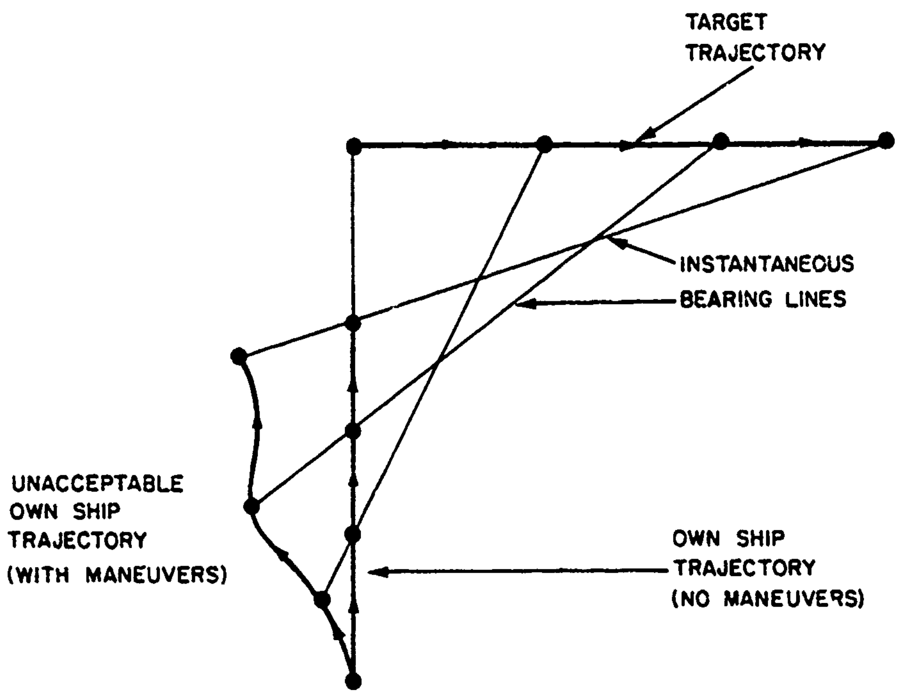

Figure 2

Another interpretation of system observability derives by recasting (19) in the form

$$\int_{t_o}^{t} (t-\tau)[a_{ox}(\tau)\cos\beta(t)-a_{oy}(\tau)\sin\beta(t)]d\tau \not\equiv 0 \tag{20}$$

and then comparing (8) with (20) to obtain an equivalent constraint relation $y(t) \not\equiv 0$. Note, however, that this latter condition further implies

$$\underline{y}(t) \not\equiv \underline{0} \tag{21}$$

which follows directly from (12). Consequently, satisfying the requirement $\det[A(t)] \not\equiv 0$ actually guarantees a <u>unique</u> and <u>non-trivial</u> TMA solution, as would be expected from physical considerations.

The preceding discussion also reveals that (20) may be utilized in lieu of (19) as an alternative test for observability. Although this particular constraint involves only the scalar function $y(t)$, it should be recognized that the foregoing analysis of $A(t)$ was necessary in order to establish the crucial equivalence relationship

$$y(t) \not\equiv 0 \Leftrightarrow \det[A(t)] \equiv 0 \tag{22}$$

from which (20) is derived.

SUMMARY AND CONCLUSIONS

The observability requirements for bearings-only TMA have been rigorously established. It was shown that unique tracking solutions can be obtained if and only if own ship accelerates, subject to the constraint

$$\int_{t_o}^{t} (t-\tau)[a_{ox}(\tau)\cos\beta(t) - a_{oy}(\tau)\sin\beta(t)]\, d\tau \not\equiv 0 \, .$$

Further analysis has revealed a previously unknown class of maneuvers for which the system remains unobservable. The bearing measurements associated with these maneuvers are indistinguishable from those corresponding to unaccelerated motion.

It is perhaps noteworthy that in realistic tracking applications own ship motion usually consists of constant velocity segments interspersed with maneuvers. Fortuitously, such tactics implicitly ensure that the observability requirements will be satisfied. The one exception is when velocity changes continually produce a zero bearing rate; i.e., the condition of "matched speed across the bearing line." Thus, while the analytical results presented here may have only limited impact on practical maneuver strategies, they nevertheless provide a comprehensive exposition of the underlying mechanism governing TMA observability.

As a final note, two situations can arise that are more applicable to these results. The first is at long target ranges where the bearing rate is often sufficiently small such that $\det[A(t)] \approx 0$ for all values of t. The second condition is the case of sparse or intermittent data. Under such circumstances, the observability requirements indicate that additional precaution should perhaps be taken in the selection of an appropriate sequence of own ship maneuvers.

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